DIFFUSION FROM THIN LAYER INTO TWO SEMIINFINITELY LARGE BODIES WITH DIFFERENT CHARACTERISTICS

G. A. Babushkin

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A solution is obtained to the problem of diffusion (heat conduction) from an infinitesimally thin layer (momentary source) into two different semiinfinitely large bodies. This solution is used for calculating how much diffusion contributes to the strength of bond between a powder coating and a substrate.

A special study of diffusion (heat conduction) from a thin layer at the boundary between two bodies with different characteristics is necessary for the solution of several important scientific and technical problems. These problems include, specifically, diffusion of vacancies in atoms adsorbed at the surface of bodies after contact has been established, diffusion of strong magnetic or orther field pulses, transfer of heat after it has been momentarily generated (for instance, during deformation of regions directly adjacent to the surfaces making contact), etc. The study of processes in powder metallurgy recently undergoing an intensive development, namely the study of sintered powder materials, of powder coatings, etc. confirms the importance of this problem.

Diffusion phenomena are encountered essentially in situations involving bodies with different characteristics. Meanwhile, the problem of diffusion (heat conduction) has been solved only for identical bodies on both sides of the boundary [1]. A solution for the case of different bodies cannot be obtained by generalization. A special solution is required here, taking into account the particular conditions. Such a solution is not only important for practical applications but also of interest mathematically.

1. Fundamental Equation. The problem of diffusion from an infinitesimally thin layer (momentary source emitting an amount q of diffusible substance from a surface of unit area in the plane x = 0 at the instant of time t = 0) into two bodies with different diffusion characteristics is solved on the basis of the equation

$$\frac{\partial c}{\partial t}=\frac{\partial}{\partial x}\left(D\frac{\partial c}{\partial x}\right),\ -\infty<x<\infty,\ t>0 \tag{1}$$

for the conditions

$$c(x,\ 0)|_{x\neq 0}=0;$$
$$c(x,\ t)_{x\to-\infty}=c(x,\ t)_{x\to+\infty}=0, \tag{2}$$

$$\int\limits_{-\infty}^{\infty} c(x,\ t)\,dx=q,\ \ t>0, \tag{3}$$

where c(x, t) is the concentration of the diffusible substance. The feature which distinguishes this problem from the problem with identical bodies on both sides of the boundary are the conditions

$$D=D_-(x<0),\ D=D_+(x>0); \tag{4}$$
$$c(x,\ t)|_{x\to-0}=c(x,\ t)|_{x\to+0}, \tag{5}$$
$$\int\limits_{-\infty}^{0} c_-(x,\ t)\,dx+\int\limits_{0}^{\infty} c_+(x,\ t)\,dx=q,\ t>0 \tag{6}$$

(subscripts "–" and "+" refer to bodies on the $x \leqslant 0$ side and on the $x \geqslant 0$ side respectively).

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2. Solution. It is obvious, on the basis of physical considerations, that the solution to the system of equations (1)-(6) must depend simultaneously on both diffusion coefficients D_- and D_+. It can be demonstrated that, after rather tedious calculations, the solution will be

$$c_\pm(x,\ t,\ D_-,\ D_+) = f_\pm(D_-,\ D_+)\, t^{-1/2} \exp[-x^2 F_\pm(D_-,\ D_+)\, t^{-1}]. \tag{7}$$

Here the upper sign and the lower sign in the subscript refer to the body in the positive half-plane and the body in the negative half-plane, respectively. Inserting solution (7) into Eq. (1) yields

$$F_\pm = 4^{-1} D_\pm^{-1}, \tag{8}$$

which indicates that the exponential part of the concentration of the diffusible substance depends only on the diffusion coefficient for the body with the concentration distribution c(x, t). Considering that

$$\Phi(\infty) = 2\pi^{-1/2} \int_0^\infty e^{-\xi^2} d\xi = 1$$

(where $\Phi(z)$ is the probability integral [1, 2]) and inserting solution (7) with expression (8) into condition (6), we find

$$f_\pm = q\pi^{-1/2} (D_-^{1/2} + D_+^{1/2})^{-1} \tag{9}$$

Inserting now expressions (8) and (9) into solution (7) yields the solution to our problem

$$c_\pm(x,\ t) = \pi^{-1/2} q\, (D_-^{1/2} + D_+^{1/2})^{-1}\, t^{-1/2} \exp(-4^{-1} x^2 D_\pm^{-1} t^{-1}). \tag{10}$$

For a physical interpretation of the result and its features, it will be convenient to consider the concentration distribution as a function of the x-coordinate at the same instants of time for a body with diffusion coefficient D_+ when it is contiguous to an identical body and when it is contiguous to a different body with diffusion coefficient D_- (say $D_- < D_+$, for specificity). The distribution curve for the second case lies above that for the first case, the ratio of ordinates being $2D_+^{1/2}(D_-^{1/2} + D_+^{1/2})^{-1}$. A decrease of the diffusion coefficient in one body results in an increase of the concentration in the other one. The concentration gradient of the diffusible substance changes in the same ratio.

3. Application of Results to Calculation of Strength of Bond between Powder Coating and Substrate. One of the most important problems in powder metallurgy is producing a strong bond between powder coating and substrate. A bond is established in two stages, diffusion and subsequent reaction. On the basis of the preceding results, let us evaluate the strength produced by diffusion.

We introduce a vacancy plane (more precisely, a layer of thickness $2^{-1}n(a_- + a_+)$, where a is the lattice parameter and $1 < n < 2$) into the contact region between two contiguous media. This corresponds, approximately, to absence of a bond at the initial instant of time. The drift of a vacancy from the vacancy plane and its replacement by an atom constitutes formation of a bond. Let N_0 be the number of vacancies in the layer at time t = 0 and N be their number in that layer at time t. Then the number of atoms in that layer at time t will be $N_0 - N$ and the relative bond strength (compared with the maximum possible bond strength σ_0) can be expressed as

$$\sigma\sigma_0^{-1} = 1 - NN_0^{-1}. \tag{11}$$

In this expression

$$N = \int_{-2^{-1}na_-}^{2^{-1}na_+} c\,dx = \int_{-2^{-1}na_-}^{0} c_- dx + \int_{0}^{2^{-1}na_+} c_+ dx, \tag{12}$$

with $c_\pm$ defined by relation (10).

Inserting expressions (12) and (10) with $q = N_0$ into relation (11) yields, after appropriate transformations,

$$\sigma\sigma_0^{-1} = 1 - D_-^{1/2}(D_-^{1/2} + D_+^{1/2})^{-1}\Phi(4^{-1}na_- D_-^{-1/2}\, t^{-1/2}) - D_+^{1/2}(D_-^{1/2} + D_+^{1/2})^{-1}\Phi(4^{-1}na_+ D_+^{-1/2}\, t^{-1/2}).$$

This expression simplifies greatly for the extreme case of short time t.

Let us now calculate the relative bond strength in the case, for example, of chromium powder deposited on iron (or between chromium and iron powders sintered together) at a contact temperature of 900°K. Considering that the diffusion coefficient $D_{\pm}$ for vacancies in iron (D_+) and chromium (D_-) is $D_{\vartheta_{\pm}} \approx D_{0_{\pm}} \exp(-\Delta H^{m}_{\vartheta_{\pm}} R^{-1} T^{-1})$ [3] (D_0 being a preexponential factor, ΔH^{m}_{ϑ} denoting the enthalpy of vacancy movement, related to the energy of diffusion activation E through the near-equality $\Delta H^{m}_{\vartheta} \approx 2^{-1}E$, and R denoting the universal gas constant), with $D_{0+} = 1.8 \cdot 10^{-5}$ m²/sec, $D_{0-} = 1.5 \cdot 10^{-8}$ m²/sec, $E_+ = 2.71 \cdot 10^{5}$ kJ/kmole, and $E_- = 2.21 \cdot 10^{5}$ kJ/kmole [3, p. 39], $n \sim 1.5$ and $a_+ \approx a_- \sim 2.5 \cdot 10^{-10}$ m [4], and $t \sim 10^{-5}$ sec (effective time of interaction in the contact region), we obtain with the aid of the table of probability integral [5, p. 129] $\varepsilon = \sigma\sigma_0^{-1} \sim 0.8$ for the relative bond strength. Assuming that σ_0 is equal to the adhesion energy for iron and chromium [6, p. 597], we obtain for the absolute bond energy $\sigma \sim 3.47$ J/m².

NOTATION

c, concentration of diffusible substance; x, coordinate normal to the boundary; t, time; D, diffusion coefficient; ε, relative bond strength (energy); E, energy of diffusion activation; T, temperature; and H, enthalpy.

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HEAT AND MOISTURE EXCHANGE OF NEWLY DENUDED ROCK MASSIF WITH A CHAMBER OF AN UNDERGROUND BUILDING

O. A. Kremnev, V. Ya. Zhuravlenko,
V. A. Shelimanov, and E. M. Kozlov

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The article presents formulas describing the fields of temperature and potential of moisture transfer in a massif, and also the dependences for calculating the heat and moisture flow from the massif into the air.

Exhausted underground spaces are widely used at present as stores, production spaces and premises for medical treatment, etc., and it becomes necessary to maintain certain temperature and moisture conditions in them. These disused workings very often have the shape of polyhedrons: parallelepipeds, prisms, etc. Existing methods of temperature and moisture calculation of cylindrical excavations [1, 2] cannot be used in similar cases. It is therefore expedient to examine the processes of heat and moisture exchange of air and a semibounded massif through a plane surface (wall).

It is known [2] that the processes of heat and mass exchange attain their greatest intensity in a newly denuded massif when the bulk of the moisture enters the air upon evaporation from the walls. In that case it may be assumed that the criterion of phase transformation in the massif is close to zero ($\varepsilon \approx 0$) and the differential equation of heat and moisture transfer had the form [3]

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